Do I Count?

Stories from Mathematics

Günter M. Ziegler

CRC Press
Taylor & Francis Group
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Originally published in German as *Darf ich Zahlen?: Geschichten aus der Mathematik* © 2010 by Piper Verlag GmbH, Müchen

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

Version Date: 20130424

International Standard Book Number-13: 978-1-4665-6492-3 (eBook - PDF)

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Preface

What does it mean, "to do mathematics"?

"Let me put up a few numbers to make the discussion more concrete" is a wonderful sentence that all regular talk-show economists should have in their arsenal. I even wanted to use it as the title of this book, but my publisher countered with the argument (though it seemed specious to me) that it was too long for a book title. We compromised on the shorter version you see on this book.

Put up some numbers—and then what? Then they are up. Just so.

I could also have called the book *What Is Mathematics?* but that is already the title of a book by Richard Courant and Herbert Robbins, a book about which one could say a few things. For example, that Courant selected the rather catchy title on the recommendation of Thomas Mann. Aside from the fact that Mann is no longer available to help with my title selection, I also did not want to write a classic mathematics book like the one by Courant and Robbins, in which mathematics is displayed along with mathematical concepts, ideas, considerations, research, and results.

My book is to be about the *doing* of mathematics, the *making* of mathematics. That is something entirely different. Try saying "love" instead of "mathematics." So what is love? What is making love? Of the latter we have some pretty concrete ideas, even if, in German, is comes across as a bad translation. My book is to be about the people behind the numbers and the places where mathematics is made. It should be about the battles for precision; about perseverance, errors, and the love of detail; about great emotions—and also about the problems that make the fight worthwhile, about recognition and prizes. This book is a trip into the world of mathematicians. This is no separate secret world. The world of mathematics is *our* world. Mathematics is not at all something distant, strange, and abstract that one can only learn about, and learn to hate, in school. Instead, it designates the corner pennants and goal lines of a playing field across which we move quite unencumbered, a field that we don't first encounter when we learn how to count, one, two, three. Are you aware that mathematics accompanies us from the very first knot in a shoelace until the artificial knee? There is mathematics in housekeeping, in communications, in traffic, and in weather reports (especially when they are accurate).

The world of mathematicians is also nothing foreign. You will find in this book all the rubrics familiar from magazines and daily papers, because they are interesting: celebrities, history, travels, politics, science and technology, weather, clever puzzles, a look into the future, and not a lot of esoterica. For which points to emphasize, I have of course oriented myself toward my own interests, preferences, and aversions by painting, in a manner of speaking, a picture of this fascinating and far-reaching world of mathematics and the making of mathematics *as I see it*. Think of it as an adventure trip with a personal tour guide. In that sense: welcome aboard!

Chapter 1

On the Number Line

"All is number," the motto of the Pythagoreans, designates the belief that the regular behavior of the world can be grasped and expressed with numbers. We still believe that today, and not without reason.

But if numbers are so fundamental, then we have to allow ourselves to ask, "What are numbers, and what are they good for?" The question may seem stupid or naive, but it is neither. The number theorist Richard Dedekind formulated the question in this way as the title of his famous book *Was sind und was sollen die Zahlen?* And the question has no simple answer. As indeed it cannot; it may well be that the regularities in the world can be phrased and understood in terms of numbers, but if so, then "one, two, three" will certainly not suffice. In fact, the so-called *natural* numbers (one, two, three, and so forth), which seem to us so concrete and obvious, already present problems—philosophical problems, of course, but also very concrete problems.

Therefore, once again, what are numbers? Are they something that designates a quantity? In that case, is $\frac{1}{2}$ a number? Or –1? Or $\sqrt{2}$? Something with which one can count? In that case, is "infinity" a number? Something with which one can compute? A domain in which one can solve equations? In that case, the "imaginary unit" $i = \sqrt{-1}$ is a number. And, as if the question were not already unclear enough, it seems as if mathematicians can't get enough, they are always creating new numbers—or will they at some point finally be satisfied?

3—Can Bees Count?

In January 2009, many newspapers around the world reported the news that "Bees can count to three." Researchers at the Bee Group of the University of Würzburg were said to have determined that bees could indeed count to three.

However, I also remembered seeing a headline not too long before that bees can count to four. What was going on? In this case it was not my bad memory, since the much more complete and thorough memory of Google confirmed that "Bees can count to four" was reported in October 2008 by *Netzeitung* (www.netzeitung.de), for example.

Let us leave aside for the moment that we find it extremely surprising that bees can count at all, never mind whether it is up to three or up to four. And let us leave aside any mysterious auras that surround these numbers. Three, like each of the smaller integers, is freighted with all sorts of symbolism, including, for example, the Christian Trinity, which involves the belief that the Father, Son, and Holy Ghost are together *one* God, or in simple terms, "one equals three." This opens up all sorts of avenues for further discussions, but that will not be our theme. Three is for us nothing but a number—but what does that mean?

The two news reports were referring to different experiments by different teams of bee researchers. In the first experiment, bees were trained to fly toward panels that showed objects, where they were rewarded with sugar water. The bees learned that it was only panels with three objects that led to food, not panels with four or six objects. They were able to make this distinction whether the panels had apples, flowers, or red or black spots. The researchers concluded that the bees had been able to form an "abstract" notion of the number 3 and could differentiate it from 4. They could, after training, fly toward panels with three objects instead of panels with four, five, or six objects. The bees could, however, not be trained to prefer panels with four objects to those with five objects, which led the researchers to conclude that bees cannot distinguish 4 from 5 nor 5 from 6. Thus the report that bees can count to three. Quite an achievement for a little animal with a brain the size of a sesame seed. Chimpanzees and humans can recognize four objects at a glance, but no more. With five or six objects it has been shown not to be possible to see "at a glance" how many there are; for that many objects, one has to start counting.

Do we now have to imagine the clever bee Maya and her somewhat duller friend Willy who mutter "one, two, three…" to themselves and point their fingers in the air? But we know bees don't have fingers. And they probably mumble only in animated television series.

As for the second report, to make bees count, one could try the following experiment: let bees fly through a tube with marks on the side and try to train them to look for food after the third mark. The marks could be put at different distances each time, so that the bees would be prevented from looking for the food at some particular distance. And indeed, the bees can learn to count to three, that is, to fly to the third marker. They can also be trained to count to four, that is, to fly to the fourth marker. But farther than that they cannot count, not even after very patient training.

Thus it came to be that in German newspapers it was reported that "bees can count to four"—and perhaps raising the question of whether journalists can count at all in a reader's mind a few weeks later when one saw the headline "Bees Can Count to Three." In any case, Professor Srinivasan, one of the investigators, reports that there are slower learners and faster learners among the bees, that is, cleverer and duller bees. But we knew that already, and we like Willy anyway.

So, do the bees then know what the number 3 *really* is? That is a fruitful philosophical question that we cannot leave entirely to the bees. However, among the reliable and firm foundations of mathematics is, naturally, an expectation that mathematicians will have neat and clear answers and concepts for such questions. They have them, too, but not for as long as one might think: this is not one of the questions answered in the depths of time or even in some Greek dialog from classic times. It was only toward the end of the nineteenth century that Georg Cantor clarified the difference between cardinal and ordinal numbers in the course of his investigations of set theory. The former describes the size of a set (a set can have one, two, three, or more elements; one is concerned with the quantity). The latter arises in the sorting of elements and then in counting them off (in which position is an element in a sequence?). There is an enormous difference, even when bees are counting. Only the journalists (and the headline writers) missed the distinction. It is, after all, not simple.

Even so, should we be impressed that bees can count to four? Actually, no. Much more impressive is the "waggle dance" in which bees dance in a geometric pattern to tell their hive mates the location of food sources. It's a powerful dance: the angle from the vertical of the line along which the bee waggles indicates the angle from the direction to the sun in which the bees must fly to find the buffet. Clearly, bees are inclined more to geometry than to arithmetic. As you can see, mathematics is rich and varied enough that all can use their talents.

5—Can Chickens Compute?

Another one of these headlines that seems to undermine our presumed supremacy in the realm of mathematics: "Chicks Can Compute—At Least Up to Five." An Italian researcher named

Rosa Rugani and her colleagues found this out. The news was circulated around the world on April 1, 2009, occasionally with an explicit comment that this was *not* an April Fool's joke. The BBC website published the result with the provocative headline "Baby Chicks Do Basic Arithmetic" and added appropriately adorable pictures of baby chicks. The scientific publication supporting the claim, according to the report, appeared in the renowned *Proceedings of the Royal Society B: Biological Sciences.*

In fact, I was able to find, with help of Google, the appropriate page, also published online on April 1. There it is claimed that freshly hatched chicks can compute, for example, "2 + 3 = 5" in their heads (where else?), which Rosa Rugani was able to confirm through a tricky set of experiments.

I was skeptical—and disturbed. The little chicks can do math? Is this an April Fool's joke after all? Suppose we take it seriously, which would mean that at least primitive arithmetic operations such as addition are not only child's play but even something freshly hatched chicks can do. But what would the little fuzzy beasts gain thereby? Some evolutionary advantage? I am definitely of the belief that intelligence has brought us (!) some (?) evolutionary advantages (although this is far from proven)—but chicks?

So I wrote on my blog "Mathematik im Alltag" (www.wissenlogs.de) with a headline "Chicks Can Compute? Help!" and asked readers for clarification and explanation. The first response was from a reader who was quite certain that Ms. Rugani conducted only serious research that was contributing further proof that chickens are smarter than we think. The reader's name was Martin Huhn (but if my blog were in English, he probably would have signed himself Martin Chicken).

And then I remembered that already in the 1980s a certain Luigi Malerba from Italy had reported:

> A learned hen wanted to teach her colleagues to count and to add. So she wrote the numbers 1 through 9 onto

> one of the walls of the chicken coop and explained that one can get even larger numbers by combining these. To teach the others addition, she wrote on the next wall: 1 + 1 = 11, 2 + 2 = 22, 3 + 3 = 33, and so forth, until 9 + 9 = 99. The hens learned to add and found it quite useful.

This clarifies everything.

10—And the Name of the Rose

A friend, and proud father, recently told me that his two-year-old son could already count to five. Only, he does not like two, so he counts, "one, (short pause), three, four, five." Now we have to clarify just what he means by "three," and we better do it soon, before he *really* learns to count.

The same problem occurs elsewhere: most airplanes have no row to which the label "13" is attached; of course, the passengers in the row labeled "14" are nonetheless sitting in the thirteenth row (and we hope they feel safer because the row is mislabeled). In many theaters and opera houses, the pleasure of having cadged a seat in the first row evaporates when one finds that one is sitting seven rows from the stage, with heads from rows A, B, C, etc., or even AA, BB, … blocking the view. (Sometimes this is a good thing—for example, if the stage is so high that the people sitting in what is really the first row need chiropractic assistance after the performance.) Or think of Douglas Adams's science fiction trilogy *Hitchhiker's Guide to the Galaxy*. The cover of the fifth (!) volume included a note that this is a book that "gives an entirely new meaning to the concept of trilogy." That it does.

The redefinition of numbers is an everyday phenomenon that we trip over everywhere. Nonetheless, the father's concern about his son's "wrong counting" is noteworthy. Of course, one could count differently, that is, use different words for the numbers. After all, we have an unmovable notion of the number 7 and of "counting to 7" that is completely independent of what we call the number, say, ☺ ☺ ☺ ☺ ☺ ☺ ☺. But why do we name the numbers as we do? And are the names we give the numbers at all important? "What's in a name? A rose by any other name would smell as sweet," says Juliet about her Romeo. Is that true for numbers, too? A brief look at history at least shows us that our numbering scheme, the Indo-Arabic positional notation with base 10 (and an additional complication arising from an occasional switching of numbers when speaking) is not at all self-evident and indisputable and not without alternatives.

The Story of the Zero

The "discovery of zero" seems like a little trifle, but it is in fact an important achievement of civilization with implications perhaps as dramatic as the discovery of the Americas (by Columbus in 1492) or of penicillin (by Alexander Fleming in 1928), even if we don't know who discovered zero or when. In fact, zero was apparently discovered at least three times in the course of history, and each time with a different meaning: the Babylonians around 700 BCE used a symbol of three ticks as a place holder; the Olmecs and, later, the Maya in Central America used (long before Columbus) a zero mark in their calendar and later in their base-20 number system; and, lastly, in India in the fifth century. There the zero was used not only as a numeral and as a placeholder but also as a number with which one could *compute*. Accordingly we must thank some anonymous Indian scholar for our positional number system, in which "2001" designates a number that is the sum of two thousands, zero hundreds, zero tens, and one unit—a very different number from 201 and 21.

This Indian positional number system subsequently came (with modifications of the symbols used for the numerals, but without modification of the principle) to Europe via the Arab–Islamic trading routes. According to legend, the mathematician Gerbert de Aurillac (c. 945–1003) played an important role in the transfer after he became Pope Sylvester II in 999, but that is unlikely, because the pope, like everyone else in Europe at the time, used an abacus to crunch his accounts. In the twelfth century, the Indo-Arabic number system first entered Western Europe through the translation of an Arabic book on computation. Widespread adoption, however, came only after the publication in 1202 of the influential *Liber Abaci* by Leonardo of Pisa, also known as Fibonacci (c. 1170–1240). Among the "common folk" in Germany, the base-10 system, including zero, became widely accepted only at the beginning of the sixteenth century. Adam Ries (c. 1492–1559) ran a school for computing in Saxony, which was subsequently continued by his sons. His second textbook, *Rechnung auff linihen und federn* (*Computation with Lines and Feathers*), taught computation not only with an abacus ("lines") but also with Indo-Arabic numerals (written with "feathers," that is, pens). The book was a remarkable bestseller, with at least 120 editions, and was used as a textbook well into the seventeenth century. Ries's name has become a byword for Germans who "know their math," for whom 120 + 69 makes, "from Adam Ries," 189.

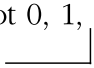
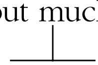
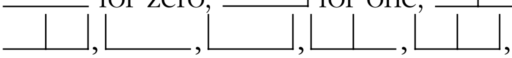
The positional system allows us to write large numbers without much effort—and certainly far less effort than the Roman numerals it replaced. Although using MMI instead of 2001 seems like a minimal effort, the federal budget, with its millions and billions of dollars, would require a lot of ink: a billion is a million thousands, which means the Romans would have had to write it as:

MMM MMM MMM MMM MMM MMM MMM MMM MMM MMM MMM MMM MMM

with a million Ms written out. Not very practical.

Apropos of large numbers: when it was discovered that a minor functionary in a French bank had gambled €50 billion and pilfered a few for himself, my daily newspaper, the Berlin *Tagesspiegel*, reported that "if one wanted to write this unbelievably large number here, the last zero would presumably be somewhere in the classifieds." By which was meant, presumably, that the number for 50 billion would require some fifteen pages to write out. Of course that's not the case, not even if one uses Roman numerals. Apparently the *Tagesspiegel* has difficulties with large numbers. But this is a tradition. There have been finance ministers in Germany who were hard-pressed to say how many zeroes a billion actually has. Even in the course of an election campaign, the finance minister then in office managed to confuse billions with trillions while the TV cameras were on.

Base 10

Of course, ten fingers. Never mind that I know mathematicians with nine fingers. One could just as easily ignore the thumbs and count with a basis of 8 or of 4. And in fact, there are Indians in South America, for example, who do just that. Or one could include fingers and toes, for a basis of 20. Even base 12 was once widespread; its vestiges can still be found in our counts of hours and months. The Babylonians computed with a base of 60. (This is impractical for computations: the "small" multiplication table is impossibly large!) Electronic computers use a binary notation, that is, a basis of 2, which is practical for them. The nicest numerals I know were designed for a base-8 system, called "octomatics." The numerals are not 0, 1, 2, etc., but much more systematic: for zero,  for one,  for two, and then  for three through seven. This is easy to understand and easy to remember (a line on the right is one, one in the middle is two, and one on the left